

R-parity violating supersymmetric Barr-Zee type contributions to the fermion electric dipole moment with weak gauge boson exchange

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The contribution of the *R*-parity violating supersymmetric model to the fermion electric dipole moment at the two-loop level is analyzed. We show that in general, the Barr-Zee type contribution to the fermion electric dipole moment with the exchange of *W* and *Z* bosons is not small compared to the currently known photon exchange one with *R*-parity violating interactions. We will then give new upper bounds on the imaginary parts of *R*-parity violating couplings from the experimental data of the electric dipole moments of the electron and of the neutron.

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I. INTRODUCTION

The standard model (SM) of particle physics, although being very successful in interpreting many experimental data, has difficulty in explaining some phenomena such as the matter abundance of our Universe. The SM has therefore to be extended.

There are currently many approaches to search for new physics (NP) beyond the SM. Among many others, the measurement of the electric dipole moment (EDM) is of particular interest. The reasons are the following. The EDM is an observable sensitive to the violation of the parity and the time-reversal symmetry (or equivalently the *CP*). The contribution from the SM is in general very small [1], and this fact makes the EDM a sensitive observable to NP with large *CP* violation. The experimental data available are very accurate for a variety of systems such as the neutron ($d_n < 2.9 \times 10^{-26} e\text{cm}$) [2], the ^{205}Tl ($d_{\text{Tl}} < 9 \times 10^{-25} e\text{cm}$) [3] and the ^{199}Hg atoms ($d_{\text{Hg}} < 3.1 \times 10^{-29} e\text{cm}$) [4], the YbF molecule (which gives bound on the electron EDM: $d_e < 1.05 \times 10^{-27} e\text{cm}$) [5], the muon [6], etc. The EDM is therefore a very good probe of NP. New generation of experiment using storage rings is also in preparation, aiming at the measurements of the EDMs of muon, proton and deuteron [7].

On the theoretical side, the minimal supersymmetric standard model (MSSM) is known to be the leading candidate of NP [8]. A general supersymmetric extension of the SM allows baryon number or lepton number violating interactions, so we must impose the conservation of *R*-parity ($R = (-1)^{3B-L+2s}$) to forbid them. This assumption is, however, completely *ad hoc*, so the *R*-parity violating (RPV) interactions have to be investigated phenomenologically. Until now, many RPV interactions were constrained by high energy experiments, low energy precision tests, and cosmological observations [9]. Thanks to many efforts in EDM experiments, many phenomenological analyses of models of multi-Higgs [10–13], supersymmetry with [14–18] and without [19–27] *R*-parity in-

variance were done, and many *CP* phases have been constrained so far.

Previous analyses showed that RPV interactions contribute to the fermion EDM starting from the two-loop level [20], and that the Barr-Zee type two-loop level diagrams give the leading effect [21, 25]. It was argued that the fermion EDM receives the largest part from the photon exchange Barr-Zee diagram, and that the other Barr-Zee type diagrams with *W* and *Z* bosons are sub-leading, on the base of the analogy between the *R*-parity violation and models with charged Higgs boson [12, 21]. We must be careful however to the fact that these two processes cannot be described similarly, since the $SU(2)_L$ gauge structure (chirality structure) of the Yukawa interactions with charged Higgs boson differs from that of the *R*-parity violation. As the chirality structure inside the fermion loop of the Barr-Zee type diagram is different, the naïve analogy no longer works. Moreover, the RPV Barr-Zee type diagram with *W* boson exchange changes the flavors of the fields in the loop, and consequently many additional combinations of RPV couplings with new flavor structure may be constrained from the the EDM experimental data.

The main purpose of this paper is then to evaluate the two-loop level Barr-Zee type diagram with *W* and *Z* boson exchange within *R*-parity violation, and to update the constraints on the RPV interactions provided by the current experimental data of the EDMs of the YbF molecule, the neutron and the ^{199}Hg atom.

This paper is organized as follows. We first present the RPV interaction in the next section. We then present and formulate in Section III the two-loop level Barr-Zee type diagram with *Z* boson exchange. In Section IV, we derive the formula for the RPV Barr-Zee type contribution with *W* boson exchange. This will be done in two steps, first by constructing the inner fermion loop. We then attach this loop to the external fermion line. In Section V, we analyze the limits provided by the current experimental data of the YbF molecule, the neutron and the ^{199}Hg atom. We finally summarize our work in the last section.

II. RPV CONTRIBUTION

The superpotential of the RPV interactions relevant in this discussion can be written as follows:

$$W_{\mathcal{R}} = \frac{1}{2} \lambda_{ijk} \epsilon_{ab} L_i^a L_j^b (E^c)_k + \lambda'_{ijk} \epsilon_{ab} L_i^a Q_j^b (D^c)_k + \frac{1}{2} \lambda''_{ijk} (U^c)_i (D^c)_j (D^c)_k, \quad (1)$$

with $i, j, k = 1, 2, 3$ indicating the generation, $a, b = 1, 2$ the $SU(2)_L$ indices. The $SU(3)_c$ indices have been omitted. L and E^c denote the lepton doublet and singlet left-chiral superfields. Q , U^c and D^c denote respectively the quark doublet, up quark singlet and down quark singlet left-chiral superfields. The bilinear term has been omitted in our discussion. We also neglected the soft breaking terms in the RPV sector. Also baryon number violating RPV interactions (λ''_{ijk}) will be omitted from now, to avoid rapid proton decay. This RPV superpotential gives the following lepton number violating Yukawa interactions:

$$\begin{aligned} \mathcal{L}_{\mathcal{R}} = & -\frac{1}{2} \lambda_{ijk} \left[\tilde{\nu}_i \bar{e}_k P_L e_j + \tilde{e}_{Lj} \bar{e}_k P_L \nu_i + \tilde{e}_{Rk}^\dagger \bar{\nu}_i^c P_L e_j \right. \\ & \left. - (i \leftrightarrow j) \right] + (\text{h.c.}) \\ & -\lambda'_{ijk} \left[\tilde{\nu}_i \bar{d}_k P_L d_j + \tilde{d}_{Lj} \bar{d}_k P_L \nu_i + \tilde{d}_{Rk}^\dagger \bar{\nu}_i^c P_L d_j \right. \\ & \left. - \tilde{e}_{Li} \bar{d}_k P_L u_j - \tilde{u}_{Lj} \bar{d}_k P_L e_i - \tilde{d}_{Rk}^\dagger \bar{e}_i^c P_L u_j \right] \\ & + (\text{h.c.}) . \end{aligned} \quad (2)$$

The projection of the chirality is given by $P_L \equiv \frac{1}{2}(1 - \gamma_5)$ [and we also define $P_R \equiv \frac{1}{2}(1 + \gamma_5)$ for later use].

III. BARR-ZEE TYPE DIAGRAM WITH Z BOSON EXCHANGE

The RPV interactions contribute to the fermion EDM starting from the two-loop level. In this section, we will

give the formula for the Barr-Zee type diagram with Z boson exchange (see Fig. 1). The computation of this

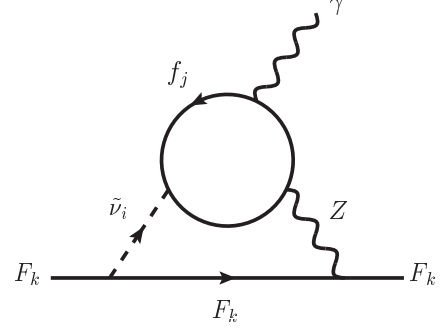


FIG. 1. An example of Barr-Zee type diagram with Z boson exchange generated with RPV interactions.

diagram is very similar to that of the Barr-Zee type diagram with photon exchange [25].

The first step of the evaluation of the Barr-Zee type two-loop diagram is to construct the gauge invariant effective $\tilde{\nu}\gamma Z$ vertex with the fermion loop diagrams, as shown in Fig. 2. This method is based on the analyses of

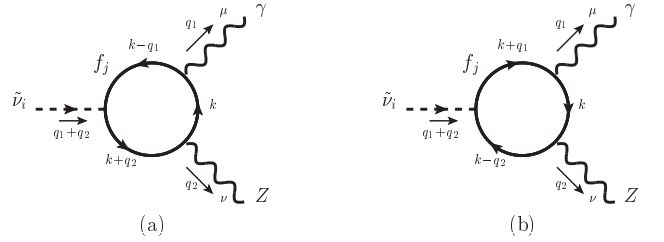


FIG. 2. One-loop effective $\tilde{\nu}\gamma Z$ vertex generated with RPV interactions.

Refs. [12, 13, 21]. We will then attach this gauge invariant effective vertex to the external fermion line to obtain the EDM operator. The amplitude of the inner fermion loop is given as

$$\begin{aligned} i\mathcal{M}_{\tilde{\nu}\gamma Z} = & -\hat{\lambda}_{ijj} n_c Q_f e^2 \alpha_f \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr}[(\not{k} + \not{q}_2 + m_{f_j})(1 - \gamma_5)(\not{k} - \not{q}_1 + m_{f_j})\gamma^\mu(\not{k} + m_{f_j})\gamma^\nu]}{[(k + q_2)^2 - m_{f_j}^2][k^2 - m_{f_j}^2][(k - q_1)^2 - m_{f_j}^2]} \\ \approx & \frac{2i}{(4\pi)^2} m_{f_j} \hat{\lambda}_{ijj} n_c Q_f e^2 \alpha_f \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \int_0^1 dx \frac{[1 - 2x(1 - x)][q_2^\mu q_1^\nu - (q_1 \cdot q_2)g^{\mu\nu}] - i\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha (q_2)_\beta}{m_{f_j}^2 - x(1 - x)q_2^2}, \end{aligned} \quad (3)$$

where the last line was given by taking the leading order contribution of the expansion in q_1 . This approximation is justified since the EDM is the first order coefficient of the multipolar expansion. Here i and j are the flavor indices of the incident sneutrino and loop fermion f , respectively, and $\hat{\lambda}$ is the RPV coupling, where $\hat{\lambda} = \lambda$

for inner loop charged lepton, and $\hat{\lambda} = \lambda'$ in the case of quark. For quark loops, the number of color is $n_c = 3$ (for lepton loop, $n_c = 1$). The weak coupling is given by α_f ($f = l, d$), where $\alpha_l \equiv \frac{1}{4}(3 \tan \theta_W - \cot \theta_W) \approx -0.065$ for the coupling of the Z boson with charged leptons, and $\alpha_d \equiv \frac{1}{12} \tan \theta_W - \frac{1}{4} \cot \theta_W \approx -0.42$ for the coupling

with quarks. We must note that α_f is the vector coupling of the Z boson with fermions, and that the axial vector coupling does not contribute.

The next step is to put the above effective $\tilde{\nu}\gamma Z$ vertex into the external fermion line to form the EDM operator. This manipulation is again very similar to that for the photon or gluon exchange Barr-Zee type diagram [25], and is given independently of the gauge as follows:

$$\begin{aligned} i\mathcal{M}_{ZBZ} &\approx i\text{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}_{ikk}^*)\frac{n_c Q_f \alpha_f \alpha_F e \alpha_{\text{em}}}{16\pi^3} \\ &\quad \times \epsilon_\mu^*(q_1)\bar{u}\sigma^{\mu\nu}(q_1)_\nu \gamma_5 u \\ &\quad \times m_{f_j} \int_0^1 dz I\left(m_Z^2, m_{\tilde{\nu}_i}^2, \frac{m_{f_j}^2}{z(1-z)}\right) \\ &\approx i\text{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}_{ikk}^*)\frac{n_c Q_f \alpha_f \alpha_F e \alpha_{\text{em}}}{16\pi^3} \\ &\quad \times \epsilon_\mu^*(q_1)\bar{u}\sigma^{\mu\nu}(q_1)_\nu \gamma_5 u \cdot \frac{m_{f_j}}{m_{\tilde{\nu}_i}^2} \ln \frac{m_{\tilde{\nu}_i}^2}{m_Z^2}, \quad (4) \end{aligned}$$

where $I(a, b, c)$ is defined as follows:

$$\begin{aligned} I(a, b, c) &= \int_0^\infty \frac{r dr}{(r+a)(r+b)(r+c)} \\ &= \frac{1}{(b-a)(c-b)(a-c)} \\ &\quad \times \left[ab \ln \left| \frac{a}{b} \right| + bc \ln \left| \frac{b}{c} \right| + ca \ln \left| \frac{c}{a} \right| \right]. \quad (5) \end{aligned}$$

In the last approximation of Eq. (4), we have used $m_{\tilde{\nu}_i}^2 \gg m_Z^2 \gg m_{f_j}^2$. Note that the fermions f_j and F_k are either down-type quark or charged lepton. The RPV coupling $\tilde{\lambda} = \lambda$ for external charged lepton, and $\tilde{\lambda} = \lambda'$ for the case of quark.

The EDM d_F is defined as follows

$$\mathcal{L}_{\text{EDM}} = -\frac{i}{2} d_F \bar{\psi} \gamma_5 \sigma^{\mu\nu} \psi F_{\mu\nu}, \quad (6)$$

where $F_{\mu\nu}$ is the electromagnetic field strength. The fermion EDM with Z boson exchange generated by RPV interactions is then given by

$$d_{F_k}^Z \approx -\text{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}_{ikk}^*)\frac{n_c Q_f \alpha_f \alpha_F e \alpha_{\text{em}}}{16\pi^3} \frac{m_{f_j}}{m_{\tilde{\nu}_i}^2} \ln \frac{m_{\tilde{\nu}_i}^2}{m_Z^2}. \quad (7)$$

Let us compare the Z boson exchange contribution with the photon exchange process. The photon exchange contribution is given by [25]

$$d_{F_k}^\gamma \approx \text{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}_{ikk}^*)\frac{n_c Q_f^2 Q_F e \alpha_{\text{em}}}{16\pi^3} \cdot \frac{m_{f_j}}{m_{\tilde{\nu}_i}^2} \left(2 + \ln \frac{m_{f_j}^2}{m_{\tilde{\nu}_i}^2} \right). \quad (8)$$

We remark that $d_{F_k}^Z$ and $d_{F_k}^\gamma$ of Eqs. (7) and (8) have the same sign. The loop factor (the logarithmic factor) has the same order of magnitude, with $\ln \frac{m_{\tilde{\nu}_i}^2}{m_Z^2}$ being around one half of $-\left(2 + \ln \frac{m_{f_j}^2}{m_{\tilde{\nu}_i}^2}\right)$ for sneutrino masses of order TeV. For lepton EDM with lepton loop, lepton EDM

with quark loop and quark EDM with lepton loop, the Z boson exchange contribution is small compared to the photon exchange one, since the weak charge of the lepton is small ($\alpha_l = -0.065$). For quark EDM with quark loop however, both diagrams have the same order of magnitude. This means that an important enhancement of the RPV contribution will occur. The detailed analysis will be done in Section V.

IV. BARR-ZEE TYPE DIAGRAM WITH W BOSON EXCHANGE

The computation of the Barr-Zee type diagram with W boson exchange is similar to that of the Barr-Zee type diagram with charged Higgs exchange, as was done in Ref. [12]. Here we should give the detail of the derivation.

A. Inner fermion loop

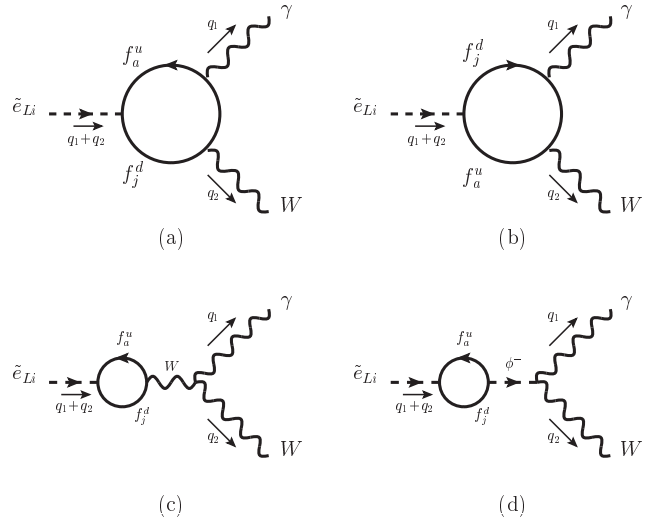


FIG. 3. One-loop effective $\tilde{e}_L \gamma W$ vertex generated with RPV interactions. Note that for the lepton loop, diagram (a) does not contribute, since the neutrino has no charge. Diagram (d) is the Nambu-Goldstone boson (ϕ) exchange contribution, which disappears in the non-linear R_ξ gauge.

As for the previously seen Z boson exchange contribution, we first derive the gauge invariant expression for the one-loop effective $\tilde{e}_L \gamma W$ vertex, generated by RPV interactions. The contributing diagrams are shown in Fig. 3. In our calculation, we have chosen the non-linear R_ξ gauge [28] which is given by the following gauge fixing (W boson)

$$\mathcal{L}_{GF}^W = -\frac{1}{\xi} |(\partial^\mu - ieA^\mu)W_\mu^+ - i\xi m_W \phi^+|^2, \quad (9)$$

In this gauge, the calculation becomes easy (interaction between Nambu-Goldstone boson (ϕ) and gauge boson

cancels). The derivation of the one-loop $\tilde{e}_L \gamma W$ amplitude goes in a manner very similar to that of the fermion-loop contribution to the charged Higgs boson decay into W boson and photon [29].

The amplitude of the one-loop effective $\tilde{e}_L \gamma W$ vertex

generated with RPV interactions is given by

$$i\mathcal{M}_{\tilde{e}\gamma W} = i\mathcal{M}_{(a)} + i\mathcal{M}_{(b)} + i\mathcal{M}_{(c)}, \quad (10)$$

where

$$i\mathcal{M}_{(a)} = \hat{\lambda}_{iaj} \frac{Q_u e^2 V_{aj}}{\sqrt{2} \sin \theta_W} n_c \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr} \left[P_L (\not{k} - \not{q}_1 + m_{f_a^u}) \gamma^\mu (\not{k} + m_{f_a^u}) \gamma^\nu P_L (\not{k} + \not{q}_2 + m_{f_j^d}) \right]}{\left[(k - q_1)^2 - m_{f_a^u}^2 \right] \left[k^2 - m_{f_a^u}^2 \right] \left[(k + q_2)^2 - m_{f_j^d}^2 \right]}, \quad (11)$$

$$i\mathcal{M}_{(b)} = \hat{\lambda}_{iaj} \frac{Q_d e^2 V_{aj}}{\sqrt{2} \sin \theta_W} n_c \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr} \left[P_L (\not{k} - \not{q}_2 + m_{f_a^u}) \gamma^\nu P_L (\not{k} + m_{f_j^d}) \gamma^\mu (\not{k} + \not{q}_1 + m_{f_j^d}) \right]}{\left[(k - q_2)^2 - m_{f_a^u}^2 \right] \left[k^2 - m_{f_j^d}^2 \right] \left[(k + q_1)^2 - m_{f_j^d}^2 \right]}, \quad (12)$$

$$i\mathcal{M}_{(c)} = \hat{\lambda}_{iaj} \frac{e^2 V_{aj}}{\sqrt{2} \sin \theta_W} n_c \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr} \left[P_L (\not{k} + m_{f_a^u}) \gamma^{\rho'} P_L (\not{k} + \not{q}_1 + \not{q}_2 + m_{f_j^d}) \right]}{\left[k^2 - m_{f_a^u}^2 \right] \left[(k + q_1 + q_2)^2 - m_{f_j^d}^2 \right]} \\ \times \frac{-1}{(q_1 + q_2)^2 - m_W^2} \left[g^{\rho\rho'} - \frac{(q_1 + q_2)^\rho (q_1 + q_2)^{\rho'}}{(q_1 + q_2)^2 - \xi m_W^2} (1 - \xi) \right] \\ \times \left[\frac{1}{\xi} (q_1^\rho + q_2^\rho) g^{\mu\nu} + g^{\nu\rho} (q_1^\mu + 2q_2^\mu) + g^{\mu\nu} (q_1^\rho - q_2^\rho) - g^{\mu\rho} (2q_1^\nu + q_2^\nu) \right], \quad (13)$$

where i, j and a denote respectively the flavor of the incident selectron, down type and up type quarks of the loop. The convention for $\hat{\lambda}$ and $\tilde{\lambda}$ are the same as for the previous case with Barr-Zee type diagram with Z boson exchange. The labels f^u and f^d denote respectively the up and down type quarks for quark loop, the neutrino and charged lepton for lepton loop. For the quark loop, the number of colors is $n_c = 3$, and $n_c = 1$ for lepton. Moreover, we have $Q_u = 0$ and $Q_d = -1$ for the lepton loop, and $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$ for the quark loop. Here V_{aj} is the CKM matrix element for the quark loop contribution. For the case of lepton loop, V_{aj} is a simple unit matrix, as we do not consider the flavor mixing in the lepton sector. It should be noted that the diagram (d) in Fig. 3 vanishes in the non-linear R_ξ gauge. The second and third lines of Eq. (13) become

$$\frac{-1}{q_{12}^2 - m_W^2} \left[g^{\rho\rho'} - \frac{q_{12}^\rho q_{12}^{\rho'}}{q_{12}^2 - \xi m_W^2} (1 - \xi) \right] \left[\frac{1}{\xi} q_{12}^\rho g^{\mu\nu} + g^{\nu\rho} (q_1^\mu + 2q_2^\mu) + g^{\mu\nu} (q_1^\rho - q_2^\rho) - g^{\mu\rho} (2q_1^\nu + q_2^\nu) \right] = -\frac{q_{12}^{\rho'}}{q_{12}^2} g^{\mu\nu}, \quad (14)$$

independently of the constant ξ , where $q_{12} = q_1 + q_2$.

The calculation of the one-loop integral can be performed by using the Passarino-Veltman one-loop tensor [30] (see Appendix A for detailed derivation). By taking the leading gauge invariant contribution first order in q_1 , we obtain the following amplitude:

$$i\mathcal{M}_{\tilde{e}\gamma W} \approx 2\hat{\lambda}_{iaj} \frac{e^2 V_{aj}}{\sqrt{2} \sin \theta_W} n_c \frac{i m_{f_j^d}}{(4\pi)^2} \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \\ \times \left[Q_u \int_0^1 dz \frac{[z(1-z)^2 - z(1-z)] [(q_1 \cdot q_2) g^{\mu\nu} - q_2^\mu q_1^\nu] - iz(1-z) \epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha (q_2)_\beta}{z(1-z)q_2^2 - (1-z)m_{f_a^u}^2 - zm_{f_j^d}^2} \right. \\ \left. + Q_d \int_0^1 dz \frac{[z^2(1-z) - z^2] [(q_1 \cdot q_2) g^{\mu\nu} - q_2^\mu q_1^\nu] - iz^2 \epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha (q_2)_\beta}{z(1-z)q_2^2 - (1-z)m_{f_a^u}^2 - zm_{f_j^d}^2} \right]. \quad (15)$$

Diagrams (a), (b) and (c) are each divergent, but the divergence of the total contribution cancels out, as is shown in Eq. (A21).

B. Second loop

The Barr-Zee type EDM with W exchange can be constructed by attaching the effective one-loop level $\tilde{e}_L \gamma W$ vertex into the second loop as shown in Fig. 4. The total

RPV Barr-Zee type contribution with W boson exchange is given by

$$\begin{aligned}
i\mathcal{M}_{\text{WBZ}} &= \hat{\lambda}_{iaj}\tilde{\lambda}_{ibk}^* \frac{n_c e^3 V_{aj} V_{bk}}{\sin^2 \theta_W} \frac{m_{f_j^d}}{(4\pi)^2} \epsilon_\mu^*(q_1) \int \frac{d^4 k'}{(2\pi)^4} \frac{\bar{u}(p-q_1) \gamma_\nu (\not{p} - \not{q}_1 - \not{k}') P_R u(p)}{[k'^2 - m_W^2] [(k' - p + q_1)^2 - m_{F_b^u}^2] [(k' + q_1)^2 - m_{\tilde{e}_{Li}}^2]} \\
&\quad \times \int_0^1 dz \frac{Q_u(1-z) + Q_d z}{z(1-z)k'^2 - (1-z)m_{f_a^u}^2 - zm_{f_j^d}^2} \\
&\quad \times \left\{ z(1-z) [(q_1 \cdot k') g^{\mu\nu} - k'^\mu q_1^\nu] - z [(q_1 \cdot k') g^{\mu\nu} - k'^\mu q_1^\nu + i\epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha k'_\beta] \right\} \\
&\quad - \hat{\lambda}_{iaj}^* \tilde{\lambda}_{ibk} \frac{n_c e^3 V_{aj} V_{bk}}{\sin^2 \theta_W} \frac{m_{f_j^d}}{(4\pi)^2} \epsilon_\mu^*(q_1) \int \frac{d^4 k'}{(2\pi)^4} \frac{\bar{u}(p-q_1) (\not{p} - \not{q}_1 - \not{k}') \gamma_\nu P_L u(p)}{[(k' + q_1)^2 - m_W^2] [(k' - p + q_1)^2 - m_{F_b^u}^2] [k'^2 - m_{\tilde{e}_{Li}}^2]} \\
&\quad \times \int_0^1 dz \frac{Q_u(1-z) + Q_d z}{z(1-z)k'^2 - (1-z)m_{f_a^u}^2 - zm_{f_j^d}^2} \\
&\quad \times \left\{ z(1-z) [(q_1 \cdot k') g^{\mu\nu} - k'^\mu q_1^\nu] - z [(q_1 \cdot k') g^{\mu\nu} - k'^\mu q_1^\nu - i\epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha k'_\beta] \right\} \\
&\approx i \text{Im}(\hat{\lambda}_{iaj} \tilde{\lambda}_{ibk}^*) \frac{n_c e \alpha_{\text{em}} V_{aj} V_{bk} m_{f_j^d}}{2(4\pi)^3 \sin^2 \theta_W} \epsilon_\mu^*(q_1) \bar{u} \sigma^{\mu\nu} (q_1)_\nu \gamma_5 u \int_0^1 dz [Q_u(1-z) + Q_d z] I\left(m_W^2, m_{\tilde{e}_{Li}}^2, \frac{m_{f_a^u}^2}{z} + \frac{m_{f_j^d}^2}{1-z}\right), \tag{16}
\end{aligned}$$

where we have kept only terms contributing to the EDM, and we have used the formula of Eq. (5). We remark that the second term in the curly brackets

$$-z [(q_1 \cdot k') g^{\mu\nu} - k'^\mu q_1^\nu + i\epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha k'_\beta]$$

in the z integration vanishes. This cancellation is reminiscent of that we encountered for the RPV Barr-Zee type EDM with exchange of photon and gluon [25].

We thus obtain the next formula for the EDM with W boson exchange:

$$\begin{aligned}
d_{F_k^d}^W &= -\text{Im}(\hat{\lambda}_{iaj} \tilde{\lambda}_{ibk}^*) \frac{n_c e \alpha_{\text{em}} V_{aj} V_{bk} m_{f_j^d}}{128\pi^3 \sin^2 \theta_W} \\
&\quad \times \int_0^1 dz (Q_u(1-z) + Q_d z) \\
&\quad \times I\left(m_W^2, m_{\tilde{e}_{Li}}^2, \frac{m_{f_a^u}^2}{z} + \frac{m_{f_j^d}^2}{1-z}\right). \tag{17}
\end{aligned}$$

We treat first the case where there is no top quark in the loop. In this case, the masses of quarks and leptons can be neglected since $m_{f_a^u}^2, m_{f_j^d}^2 \ll m_W^2, m_{\tilde{e}_{Li}}^2$. The EDM is then given by

$$d_{F_k^d}^W \approx -\text{Im}(\hat{\lambda}_{iaj} \tilde{\lambda}_{ibk}^*) \frac{s_f e \alpha_{\text{em}} V_{aj} V_{bk}}{256\pi^3 \sin^2 \theta_W} \frac{m_{f_j^d}}{m_{\tilde{e}_{Li}}^2} \ln \frac{m_{\tilde{e}_{Li}}^2}{m_W^2}, \tag{18}$$

where we have assumed that $m_{\tilde{e}_{Li}}^2 \gg m_W^2$. Here $s_f = n_c(Q_u + Q_d) = +1$ for inner quark loop and $s_f = -1$ for lepton loop. We have verified numerically that the above

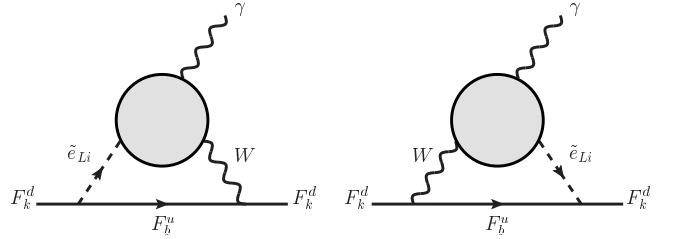


FIG. 4. Barr-Zee type contribution to the fermion EDM with W boson exchange generated from RPV interactions. The diagram on the left side involves the complex conjugated combination of RPV couplings. F^u and F^d denote respectively the up and down type quarks for quark EDM, the neutrino and charged lepton for lepton EDM.

approximation works well when top quark is absent in the loop.

For the case where top quark is present in the loop, the mass $m_{f_a^u}^2$ cannot be neglected. The integral of Eq. (17) can be performed analytically, and we obtain

$$\begin{aligned}
d_{F_k^d}^W &\approx -\text{Im}(\lambda'_{iaj} \tilde{\lambda}_{ibk}^*) \frac{e \alpha_{\text{em}} V_{aj} V_{bk}}{128\pi^3 \sin^2 \theta_W} \frac{m_{f_j^d}}{m_W^2 - m_{\tilde{e}_{Li}}^2} \\
&\quad \times \left[\left(\frac{3}{z_W^2} - \frac{2}{z_W} \right) \left(\text{Li}_2(1-z_W) - \frac{\pi^2}{6} \right) \right. \\
&\quad \left. + \frac{3}{z_W} (1 - \ln(z_W)) + \frac{1}{2} \ln(z_W) \right] - (z_W \leftrightarrow z_{\tilde{e}}), \tag{19}
\end{aligned}$$

where $z_W \equiv m_W^2/m_t^2$ and $z_{\tilde{e}} \equiv m_{\tilde{e}_{Li}}^2/m_t^2$, and $\text{Li}_2(z)$ is the dilogarithm function (for the calculation of the dilogarithm function, we have used the computational code of Ref. [31]). The integral with top quark in the loop is of course smaller than the case without it. For $m_{\tilde{e}_{Li}} = 1$ TeV, the loop integral with top quark becomes about the 30% of that without top quark.

V. ANALYSIS

A. Constraints on $\text{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}_{kk}^*)$

Let us first discuss the case where no change of generation occurs in the inner fermion loop and external fermion line (i.e. $a = j$ and $b = k$). This contribution interferes with the Barr-Zee type process with photon or gluon exchange, previously investigated in Refs. [9, 20–22, 24, 25]. This part is thus an extension of these analyses. In this analysis, we assume that all sparticle masses are equal to 1 TeV [32].

The first case to treat is the lepton EDM with inner lepton loop, where the combinations of RPV couplings $\text{Im}(\lambda_{311}\lambda_{322}^*)$ and $\text{Im}(\lambda_{211}\lambda_{233}^*)$ are involved. This contribution can be constrained by the current experimental data of the electron EDM given by the YbF molecule experiment [5]:

$$|d_e| < 1.05 \times 10^{-27} e \text{ cm} . \quad (20)$$

The Barr-Zee type lepton EDM with lepton loop receives negligible contribution from the Z boson exchange diagram, since the charged leptons have small coupling with the Z boson ($\alpha_e \approx -0.065$). The W boson exchange effect is also small compared to the photon exchange one by about one order of magnitude. We must also note that the sign of the W boson exchange Barr-Zee type EDM is the same as the photon exchange EDM. The photon exchange contribution thus dominates, as was claimed in previous works. The upper limit to the combinations of RPV couplings is shown in Table I. To our best knowledge, the experimental data of the electron EDM given by the YbF molecule experiment give the tightest limits.

The second case to discuss is the lepton EDM with inner quark loop. In this case also, the Barr-Zee type diagrams with Z boson exchange is small, due to the small weak coupling. Numerically, it is smaller than 5%, with the same sign as the photon exchange EDM. The W boson exchange contribution is however not negligible. This is because the photon exchange EDM receives a suppression by the fractional charge. For the bottom quark loop contribution, the ratio is $d_e^W/d_e^\gamma \approx 0.25$. Moreover d_e^γ and d_e^W have opposite sign. This means that the total EDM contribution becomes significantly small. By using the experimental data of Eq. (20), it is possible to constrain the combinations of RPV couplings $\text{Im}(\lambda_{i11}\lambda_{i11}^*)$ ($i = 2, 3$), $\text{Im}(\lambda_{i11}\lambda_{i22}^*)$ ($i = 1, 3$) and $\text{Im}(\lambda_{i11}\lambda_{i33}^*)$ ($i = 1, 2$). For these RPV bilinears,

TABLE I. Upper bounds to the RPV couplings given by the current EDM experimental data of the electron and neutron, with $m_{\tilde{e}_i} = m_{\tilde{e}_{Li}} = 1$ TeV. For comparison, we have also written the limits provided only by the photon exchange Barr-Zee type contribution. Limits given by other experiments [9, 26] are also shown. $i = 1, 2, 3$.

RPV couplings	This work	d^γ only	Other experiments
$ \text{Im}(\lambda_{311}^* \lambda_{322}) $	2.0×10^{-3}	2.1×10^{-3}	0.15
$ \text{Im}(\lambda_{211}^* \lambda_{233}) $	1.7×10^{-4}	1.9×10^{-4}	0.25
$ \text{Im}(\lambda_{211}^* \lambda'_{211}) $	1.2×10^{-1}	1.0×10^{-1}	7.9×10^{-8}
$ \text{Im}(\lambda_{311}^* \lambda'_{311}) $	1.2×10^{-1}	1.0×10^{-1}	7.9×10^{-8}
$ \text{Im}(\lambda_{211}^* \lambda'_{222}) $	8.4×10^{-3}	6.7×10^{-3}	3.6×10^{-6}
$ \text{Im}(\lambda_{311}^* \lambda'_{322}) $	8.4×10^{-3}	6.7×10^{-3}	3.6×10^{-6}
$ \text{Im}(\lambda_{211}^* \lambda'_{233}) $	3.2×10^{-4}	2.9×10^{-4}	1.8×10^{-4}
$ \text{Im}(\lambda_{311}^* \lambda'_{333}) $	3.2×10^{-4}	2.9×10^{-4}	1.8×10^{-4}
$ \text{Im}(\lambda_{122}^* \lambda'_{111}) $	3.0×10^{-1}	3.7×10^{-1}	4.4×10^{-2}
$ \text{Im}(\lambda_{322}^* \lambda'_{311}) $	3.0×10^{-1}	3.7×10^{-1}	6.0×10^{-3}
$ \text{Im}(\lambda_{133}^* \lambda'_{111}) $	2.5×10^{-2}	3.4×10^{-2}	2.6×10^{-2}
$ \text{Im}(\lambda_{233}^* \lambda'_{211}) $	2.5×10^{-2}	3.4×10^{-2}	2.9×10^{-2}
$ \text{Im}(\lambda_{i11}^* \lambda'_{i22}) $	1.5	1.2	3.1×10^{-4}
$ \text{Im}(\lambda_{i11}^* \lambda'_{i33}) $	3.5×10^{-2}	5.1×10^{-2}	1.1×10^{-5}

there already exist stronger constraints, given by the experimental data of the ^{199}Hg atom [4] via P , CP -odd electron-nucleon interaction [22, 33, 34]. It is not possible to give new upper limits on the corresponding RPV interactions with Barr-Zee type process.

The third case to consider is the quark EDM with lepton inner loop. The Z boson exchange contribution has a small contribution for the same reason as the previous case (numerically, less than 5%). The W boson exchange EDM is however sizable, with about 38% of the photon exchange EDM for τ loop contribution, and 24% for μ loop contribution. The quark EDM with lepton loop has the same sign as the photon exchange Barr-Zee diagram, so d^W and d^γ interfere constructively. We obtain thus tighter limits for the RPV couplings. By using the current experimental data of the neutron EDM [2]

$$|d_n| < 2.9 \times 10^{-26} e \text{ cm} . \quad (21)$$

and the relation between the neutron and quark EDMs calculated with the QCD sum rules [35, 36]

$$d_n = 0.47d_d - 0.12d_u + e(0.35d_d^c + 0.17d_u^c) , \quad (22)$$

the combinations of RPV couplings $\text{Im}(\lambda_{i22}\lambda_{i11}^*)$ ($i = 1, 3$) and $\text{Im}(\lambda_{i33}\lambda_{i11}^*)$ ($i = 1, 2$) can be constrained. The result is given in Table I. In this case we could give a new upper limits on $\text{Im}(\lambda_{i33}\lambda_{i11}^*)$ ($i = 1, 2$).

The final case to consider is the quark EDM with inner quark loop. In this case both Z and W boson exchange contributions are sizable compared with the photon exchange one. The first reason is that the photon exchange EDM receives a double suppression from the fractional charge of the down type quark. For the Z boson exchange process, the suppression due to the small weak coupling

between Z and charged lepton is absent so that d^Z and d^γ have similar size. The sign between them is the same and the interference is constructive. For the W boson, the contribution is around 35% of the photon exchange EDM. We must note that the Barr-Zee type diagram with W boson and inner quark loop act destructively against the rest. The RPV interactions contributing to the quark EDM with quark loop can be constrained by the neutron EDM experimental data. These RPV interactions are, however, already strongly constrained by the experimental data of the ^{199}Hg atom [4] via chomo-EDM and P , CP -odd 4-quark interactions [33, 35, 37]. It is not possible to give new upper limits on the corresponding RPV interactions with Barr-Zee type process.

B. Constraints on $\text{Im}(\hat{\lambda}_{iaj}\tilde{\lambda}_{ibk}^*)$

The Barr-Zee type diagram with W boson exchange provides also possibilities to constrain new combinations of RPV couplings through the generation change of the CKM matrix. By combining Eqs. (19) and (18) with the EDM experimental data (20) and (21) (with the relation (22)), we obtain upper limits on the imaginary parts of RPV couplings shown in Table II.

We see that the experimental data of the electron EDM give new upper limits on $|\text{Im}(\lambda_{211}^*\lambda_{223})|$ and $|\text{Im}(\lambda_{311}^*\lambda_{323})|$. These upper bounds could be obtained thanks to the enhancement due to the large bottom quark mass in the inner loop. Other limits on RPV couplings given by this analysis are weaker than those obtained by other experiments, but we remark that many bounds

on RPV couplings are not so different. The progress on neutron and atomic/molecular EDMs are very promising, and many combinations of RPV couplings are in the reach of the next generation experiments. The Barr-Zee type EDM with W boson exchange thus provides a very wide accessibility to the CP violation of the RPV sector.

VI. CONCLUSION

In conclusion we have discussed additional contribution to the fermion EDM in the RPV supersymmetric models. We have calculated the contribution of the two-loop level Barr-Zee type diagram with W and Z boson exchange. We have then found that these contribution are not negligible in many situations, contrary to the claim in Ref. [21]. In particular, the quark EDM with lepton inner loop was enhanced with the W boson exchange diagram and we have done a significant update of the upper limits to the RPV couplings as $|\text{Im}(\lambda_{j33}^*\lambda'_{j11})| < 2.5 \times 10^{-2}$ ($j = 1, 2$). We have also found that many additional RPV contributions generated by the flavor change due to the W boson exchange exist, with many combinations of RPV couplings. In this case it was also possible to give a new constraint on RPV couplings as $|\text{Im}(\lambda_{j11}^*\lambda'_{j23})| < 1.6 \times 10^{-2}$ ($j = 2, 3$). Moreover, many RPV couplings can potentially be constrained by near future EDM experiments, since the upper limits given by this analysis and those from other experiments have close value. The W and Z boson exchange contribution thus provides a very wide possibility to approach the CP violation of the RPV sector.

Appendix A: Passarino-Veltman one-loop integral and the $\tilde{e}_L\gamma W$ amplitude

For the calculation of the one-loop $\tilde{e}_L i\gamma W$ process (Eqs. (11), (12) and (13)), it is convenient to use the formalism of Passarino-Veltman one-loop integral [30].

Let us define the one-loop tensor as

$$B_0(p_1^2, m_0^2, m_1^2) \equiv \frac{(2\pi\mu)^\epsilon}{i\pi^2} \int d^d k \frac{1}{[k^2 - m_0^2][(k+p_1)^2 - m_1^2]}, \quad (\text{A1})$$

$$B^\mu(p_1^2, m_0^2, m_1^2) \equiv \frac{(2\pi\mu)^\epsilon}{i\pi^2} \int d^d k \frac{k^\mu}{[k^2 - m_0^2][(k+p_1)^2 - m_1^2]}, \quad (\text{A2})$$

$$C_0(p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) \equiv \frac{(2\pi\mu)^\epsilon}{i\pi^2} \int d^d k \frac{1}{[k^2 - m_0^2][(k+p_1)^2 - m_1^2][(k+p_2)^2 - m_2^2]}, \quad (\text{A3})$$

$$C^\mu(p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) \equiv \frac{(2\pi\mu)^\epsilon}{i\pi^2} \int d^d k \frac{k^\mu}{[k^2 - m_0^2][(k+p_1)^2 - m_1^2][(k+p_2)^2 - m_2^2]}, \quad (\text{A4})$$

$$C^{\mu\nu}(p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) \equiv \frac{(2\pi\mu)^\epsilon}{i\pi^2} \int d^d k \frac{k^\mu k^\nu}{[k^2 - m_0^2][(k+p_1)^2 - m_1^2][(k+p_2)^2 - m_2^2]}, \quad (\text{A5})$$

where $d = 4 - \epsilon$ is the space-time dimension shifted by ϵ . Due to the Lorentz covariance, B^μ , C^μ and $C^{\mu\nu}$ can be decomposed as follows

$$B^\mu = B_1 p_1^\mu, \quad (\text{A6})$$

$$C^\mu = C_1 p_1^\mu + C_2 p_2^\mu, \quad (\text{A7})$$

$$C^{\mu\nu} = C_{00} g^{\mu\nu} + C_{11} p_1^\mu p_1^\nu + C_{12} (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) + C_{22} p_2^\mu p_2^\nu, \quad (\text{A8})$$

TABLE II. Upper bounds to the RPV couplings given by the current EDM experimental data for $m_{\tilde{e}_{Li}} = 1$ TeV. Limits from other experiments [9, 26] are also shown.

RPV couplings	Limits given by this analysis	Other experiments
$ \text{Im}(\lambda_{211}^* \lambda'_{221}) $	2.5 (<i>e</i> EDM [5])	2.0×10^{-5}
$ \text{Im}(\lambda_{311}^* \lambda'_{321}) $	2.5 (<i>e</i> EDM [5])	2.0×10^{-5}
$ \text{Im}(\lambda_{211}^* \lambda'_{231}) $	210 (<i>e</i> EDM [5])	8.2×10^{-4}
$ \text{Im}(\lambda_{311}^* \lambda'_{331}) $	210 (<i>e</i> EDM [5])	8.2×10^{-4}
$ \text{Im}(\lambda_{211}^* \lambda'_{212}) $	0.12 (<i>e</i> EDM [5])	3.3×10^{-3}
$ \text{Im}(\lambda_{311}^* \lambda'_{312}) $	0.12 (<i>e</i> EDM [5])	3.3×10^{-3}
$ \text{Im}(\lambda_{211}^* \lambda'_{232}) $	2.1 (<i>e</i> EDM [5])	2.9×10^{-2}
$ \text{Im}(\lambda_{311}^* \lambda'_{332}) $	2.1 (<i>e</i> EDM [5])	2.9×10^{-2}
$ \text{Im}(\lambda_{211}^* \lambda'_{213}) $	0.18 (<i>e</i> EDM [5])	2.9×10^{-2}
$ \text{Im}(\lambda_{311}^* \lambda'_{313}) $	0.18 (<i>e</i> EDM [5])	1.7×10^{-2}
$ \text{Im}(\lambda_{211}^* \lambda'_{223}) $	1.6×10^{-2} (<i>e</i> EDM [5])	2.9×10^{-2}
$ \text{Im}(\lambda_{311}^* \lambda'_{323}) $	1.6×10^{-2} (<i>e</i> EDM [5])	1.7×10^{-2}
$ \text{Im}(\lambda_{122}^* \lambda'_{121}) $	6.6 (<i>n</i> EDM [2])	2.9×10^{-2}
$ \text{Im}(\lambda_{322}^* \lambda'_{321}) $	6.6 (<i>n</i> EDM [2])	2.9×10^{-2}
$ \text{Im}(\lambda_{122}^* \lambda'_{131}) $	180 (<i>n</i> EDM [2])	0.70
$ \text{Im}(\lambda_{322}^* \lambda'_{331}) $	180 (<i>n</i> EDM [2])	0.70
$ \text{Im}(\lambda_{133}^* \lambda'_{121}) $	0.39 (<i>n</i> EDM [2])	1.7×10^{-2}
$ \text{Im}(\lambda_{233}^* \lambda'_{221}) $	0.39 (<i>n</i> EDM [2])	2.9×10^{-2}
$ \text{Im}(\lambda_{133}^* \lambda'_{131}) $	11 (<i>n</i> EDM [2])	0.42
$ \text{Im}(\lambda_{233}^* \lambda'_{231}) $	11 (<i>n</i> EDM [2])	0.70
$ \text{Im}(\lambda_{i11}^* \lambda'_{i12}) $ ($i = 1, 2, 3$)	7.0 (<i>n</i> EDM [2])	7.3×10^{-4}
$ \text{Im}(\lambda_{j11}^* \lambda'_{j32}) $ ($j = 1, 2$)	130 (<i>n</i> EDM [2])	8.0×10^{-2}
$ \text{Im}(\lambda_{311}^* \lambda'_{332}) $	130 (<i>n</i> EDM [2])	1.4×10^{-2}
$ \text{Im}(\lambda_{121}^* \lambda'_{112}) $	31 (<i>n</i> EDM [2])	4.0×10^{-4}
$ \text{Im}(\lambda_{k21}^* \lambda'_{k12}) $ ($k = 2, 3$)	31 (<i>n</i> EDM [2])	3.2×10^{-3}
$ \text{Im}(\lambda_{i21}^* \lambda'_{i22}) $ ($i = 1, 2, 3$)	7.1 (<i>n</i> EDM [2])	7.3×10^{-4}
$ \text{Im}(\lambda_{i21}^* \lambda'_{i32}) $ ($i = 1, 2, 3$)	550 (<i>n</i> EDM [2])	8.0×10^{-2}
$ \text{Im}(\lambda_{i31}^* \lambda'_{i12}) $ ($i = 1, 2, 3$)	800 (<i>n</i> EDM [2])	8.0×10^{-2}
$ \text{Im}(\lambda_{i31}^* \lambda'_{i22}) $ ($i = 1, 2, 3$)	190 (<i>n</i> EDM [2])	8.0×10^{-2}
$ \text{Im}(\lambda_{i31}^* \lambda'_{i32}) $ ($i = 1, 2, 3$)	1.5×10^4 (<i>n</i> EDM [2])	7.3×10^{-4}
$ \text{Im}(\lambda_{i11}^* \lambda'_{i13}) $ ($i = 1, 2, 3$)	11 (<i>n</i> EDM [2])	3.2×10^{-3}
$ \text{Im}(\lambda_{i11}^* \lambda'_{i23}) $ ($i = 1, 2, 3$)	0.93 (<i>n</i> EDM [2])	3.2×10^{-3}
$ \text{Im}(\lambda_{i21}^* \lambda'_{i13}) $ ($i = 1, 2, 3$)	47 (<i>n</i> EDM [2])	3.2×10^{-3}
$ \text{Im}(\lambda_{i21}^* \lambda'_{i23}) $ ($i = 1, 2, 3$)	4.0 (<i>n</i> EDM [2])	3.2×10^{-3}
$ \text{Im}(\lambda_{121}^* \lambda'_{133}) $	0.53 (<i>n</i> EDM [2])	2.0×10^{-4}
$ \text{Im}(\lambda_{k21}^* \lambda'_{k33}) $ ($k = 2, 3$)	0.53 (<i>n</i> EDM [2])	8.0×10^{-2}
$ \text{Im}(\lambda_{131}^* \lambda'_{113}) $	1.3×10^3 (<i>n</i> EDM [2])	1.7×10^{-5}
$ \text{Im}(\lambda_{k31}^* \lambda'_{k13}) $ ($k = 2, 3$)	1.3×10^3 (<i>n</i> EDM [2])	8.0×10^{-2}
$ \text{Im}(\lambda_{i31}^* \lambda'_{i23}) $ ($i = 1, 2, 3$)	110 (<i>n</i> EDM [2])	8.0×10^{-2}
$ \text{Im}(\lambda_{131}^* \lambda'_{133}) $	14 (<i>n</i> EDM [2])	4.9×10^{-3}
$ \text{Im}(\lambda_{k31}^* \lambda'_{k33}) $ ($k = 2, 3$)	14 (<i>n</i> EDM [2])	2.0

where the arguments of the loop functions were omitted. The loop functions B_0 , B_1 , C_0 , C_{00} , C_{11} , C_{12} and C_{22} can be explicitly calculated, but they satisfy also many useful relations. Note also that B_0 , B_1 and C_{00} are divergent. By contracting C^μ with external momenta p_1^μ or p_2^μ , we obtain

$$\begin{aligned}
C_\mu p_1^\mu &= \frac{1}{2} B_0(p_2^2, m_0^2, m_2^2) - \frac{1}{2} B_0((p_1 - p_2)^2, m_1^2, m_2^2) - \frac{1}{2} [p_1^2 - m_1^2 + m_0^2] C_0, \\
C_\mu p_2^\mu &= \frac{1}{2} B_0(p_1^2, m_0^2, m_1^2) - \frac{1}{2} B_0((p_1 - p_2)^2, m_1^2, m_2^2) - \frac{1}{2} [p_2^2 - m_2^2 + m_0^2] C_0.
\end{aligned} \tag{A9}$$

Here we have omitted the arguments for C_0 which are implicitly the same as those of Eq. (A3).

Similarly, we obtain also additional relations by contracting $C^{\mu\nu}$ with two external momenta.

$$dC_{00} + C_{11}p_1^2 + 2C_{12}(p_1 \cdot p_2) + C_{22}p_2^2 = B_0((p_2 - p_1)^2, m_1^2, m_2^2) + m_0^2 C_0, \quad (\text{A10})$$

$$\begin{aligned} C_{00} + C_{11}p_1^2 + C_{12}(p_1 \cdot p_2) &= \frac{1}{2}B_1((p_2 - p_1)^2, m_1^2, m_2^2) + \frac{1}{2}B_0((p_2 - p_1)^2, m_1^2, m_2^2) \\ &\quad - \frac{1}{2}[p_1^2 - m_1^2 + m_0^2] C_1, \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} C_{12}p_1^2 + C_{22}(p_1 \cdot p_2) &= \frac{1}{2}B_1(p_2^2, m_0^2, m_2^2) - \frac{1}{2}B_1((p_2 - p_1)^2, m_1^2, m_2^2) \\ &\quad - \frac{1}{2}[p_1^2 - m_1^2 + m_0^2] C_2, \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} C_{11}(p_1 \cdot p_2) + C_{12}p_2^2 &= \frac{1}{2}B_1(p_1^2, m_0^2, m_1^2) + \frac{1}{2}B_1((p_2 - p_1)^2, m_1^2, m_2^2) \\ &\quad + \frac{1}{2}B_0((p_2 - p_1)^2, m_1^2, m_2^2) - \frac{1}{2}[p_2^2 - m_2^2 + m_0^2] C_1, \end{aligned} \quad (\text{A13})$$

$$C_{00} + C_{12}(p_1 \cdot p_2) + C_{22}p_2^2 = -\frac{1}{2}B_1((p_2 - p_1)^2, m_1^2, m_2^2) - \frac{1}{2}[p_2^2 - m_2^2 + m_0^2] C_2, \quad (\text{A14})$$

where again arguments for C functions were omitted. Applying all these relations to Eqs. (11), we obtain

$$\begin{aligned} i\mathcal{M}_{(a)} &= \frac{2im_{f_j^d}}{(4\pi)^2} \hat{\lambda}_{iaj} \frac{Q_u e^2 V_{aj}}{\sqrt{2} \sin \theta_W} n_c \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \\ &\quad \times \left\{ [((q_1 \cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu)(2C_{12} + C_2) + i\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha (q_2)_\beta C_2] (q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^u}^2, m_{f_j^d}^2, m_{f_j^d}^2) \right. \\ &\quad \left. + B_1((q_1 + q_2)^2, m_{f_j^u}^2, m_{f_j^d}^2) g^{\mu\nu} \right\}. \end{aligned} \quad (\text{A15})$$

Note that the expression in parenthesis $(q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^u}^2, m_{f_j^d}^2, m_{f_j^d}^2)$ denote the common arguments for the loop functions C_{12} and C_2 . For Eqs. (12) and (13), we obtain similarly

$$\begin{aligned} i\mathcal{M}_{(b)} &= \frac{2im_{f_j^d}}{(4\pi)^2} \hat{\lambda}_{iaj} \frac{Q_d e^2 V_{aj}}{\sqrt{2} \sin \theta_W} n_c \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) \\ &\quad \times \left\{ [((q_1 \cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu)(2C_{12} - C_2 - C_0) - i\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha (q_2)_\beta (C_2 + C_0)] (q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^d}^2, m_{f_j^d}^2, m_{f_j^u}^2) \right. \\ &\quad \left. + [B_1((q_1 + q_2)^2, m_{f_j^d}^2, m_{f_j^u}^2) + B_0((q_1 + q_2)^2, m_{f_j^d}^2, m_{f_j^u}^2)] g^{\mu\nu} \right\}, \end{aligned} \quad (\text{A16})$$

$$i\mathcal{M}_{(c)} = \frac{2im_{f_j^d}}{(4\pi)^2} \hat{\lambda}_{iaj} \frac{-e^2 V_{aj}}{\sqrt{2} \sin \theta_W} n_c \epsilon_\mu^*(q_1) \epsilon_\nu^*(q_2) g^{\mu\nu} B_1((q_1 + q_2)^2, m_{f_j^u}^2, m_{f_j^d}^2). \quad (\text{A17})$$

We should note that the arguments of the C loop functions for $i\mathcal{M}_{(b)}$ differ from those for $i\mathcal{M}_{(a)}$.

We should now show that terms with B functions cancel with each other for $i\mathcal{M}_{(a)} + i\mathcal{M}_{(b)} + i\mathcal{M}_{(c)}$. The loop functions B_0 and B_1 can be directly calculated using Feynman parameters as

$$B_0(q^2, m_0^2, m_1^2) = \left[\frac{2}{\epsilon} - \gamma + \ln(4\pi) \right] - \int_0^1 dx \ln \left(\frac{-x(1-x)q^2 + (1-x)m_0^2 + xm_1^2}{\mu^2} \right), \quad (\text{A18})$$

$$B_1(q^2, m_0^2, m_1^2) = -\frac{1}{2} \left[\frac{2}{\epsilon} - \gamma + \ln(4\pi) \right] + \int_0^1 dx x \ln \left(\frac{-x(1-x)q^2 + (1-x)m_0^2 + xm_1^2}{\mu^2} \right), \quad (\text{A19})$$

where γ is the Euler constant and μ the mass scale peculiar in the dimensional regularization. By noting that

$$\begin{aligned} B_0(q^2, m_0^2, m_1^2) + B_1(q^2, m_0^2, m_1^2) &= \frac{1}{2} \left[\frac{2}{\epsilon} - \gamma + \ln(4\pi) \right] - \int_0^1 dx (1-x) \ln \left(\frac{-x(1-x)q^2 + (1-x)m_0^2 + xm_1^2}{\mu^2} \right) \\ &= \frac{1}{2} \left[\frac{2}{\epsilon} - \gamma + \ln(4\pi) \right] - \int_0^1 dx x \ln \left(\frac{-x(1-x)q^2 + (1-x)m_1^2 + xm_0^2}{\mu^2} \right) \\ &= -B_1(q^2, m_1^2, m_0^2), \end{aligned} \quad (\text{A20})$$

the sum of the B functions for $i\mathcal{M}_{(a)} + i\mathcal{M}_{(b)} + i\mathcal{M}_{(c)}$ becomes

$$\begin{aligned}
& Q_u B_1((q_1 + q_2)^2, m_{f_j^u}^2, m_{f_j^d}^2) + Q_d \left[B_1((q_1 + q_2)^2, m_{f_j^d}^2, m_{f_j^u}^2) + B_0((q_1 + q_2)^2, m_{f_j^d}^2, m_{f_j^u}^2) \right] \\
& - B_1((q_1 + q_2)^2, m_{f_j^u}^2, m_{f_j^d}^2) \\
& = (Q_u - Q_d - 1) B_1((q_1 + q_2)^2, m_{f_j^u}^2, m_{f_j^d}^2) \\
& = 0.
\end{aligned} \tag{A21}$$

We see that the divergence cancels. The evaluation of the remaining C functions can also be done by direct calculation of the integrals.

$$C_{12}(q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^u}^2, m_{f_j^u}^2, m_{f_j^d}^2) = \int_0^1 dz \int_0^{1-z} dx \frac{xz}{x(1-x)q_1^2 + z(1-z)q_2^2 + 2xz(q_1 \cdot q_2) - (1-z)m_{f_j^u}^2 - zm_{f_j^d}^2}, \tag{A22}$$

$$C_2(q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^u}^2, m_{f_j^u}^2, m_{f_j^d}^2) = \int_0^1 dz \int_0^{1-z} dx \frac{-z}{x(1-x)q_1^2 + z(1-z)q_2^2 + 2xz(q_1 \cdot q_2) - (1-z)m_{f_j^u}^2 - zm_{f_j^d}^2}, \tag{A23}$$

$$C_{12}(q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^d}^2, m_{f_j^d}^2, m_{f_j^u}^2) = \int_0^1 dz \int_0^{1-z} dx \frac{xz}{x(1-x)q_1^2 + z(1-z)q_2^2 + 2xz(q_1 \cdot q_2) - (1-z)m_{f_j^d}^2 - zm_{f_j^u}^2}, \tag{A24}$$

$$C_2(q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^d}^2, m_{f_j^d}^2, m_{f_j^u}^2) = \int_0^1 dz \int_0^{1-z} dx \frac{-z}{x(1-x)q_1^2 + z(1-z)q_2^2 + 2xz(q_1 \cdot q_2) - (1-z)m_{f_j^d}^2 - zm_{f_j^u}^2}, \tag{A25}$$

$$C_0(q_1^2, (q_1 + q_2)^2, q_2^2, m_{f_j^d}^2, m_{f_j^d}^2, m_{f_j^u}^2) = \int_0^1 dz \int_0^{1-z} dx \frac{1}{x(1-x)q_1^2 + z(1-z)q_2^2 + 2xz(q_1 \cdot q_2) - (1-z)m_{f_j^d}^2 - zm_{f_j^u}^2}. \tag{A26}$$

By applying the above formulae to Eqs. (A15) and (A16), taking the first order approximation in q_1 , and integrating by x , we obtain Eq. (15).

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